

Emergent Relativity: Neutrinos as Probe of the Underlying Theory

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Abstract

Neutrinos allow for a test of the hypothesis that the fermions of the Standard Model have Fermi-point splitting, analogous to the fermionic quasi-particles of certain condensed-matter systems. If present, the corresponding Lorentz-violating terms in the Hamiltonian may provide a new source of **T** and **CP** violation in the leptonic sector, which is not directly related to mass.

1. Introduction

The basic idea of this talk is to suggest neutrinos as a probe of radically new physics. Of course, this is a long-shot ... but worth trying.

One example of such new physics would be related to the concept of *emergent symmetries* [1,2,3,4]. Lorentz invariance, for example, would not be a fundamental symmetry but an emergent phenomenon at low energies.

In order to be specific, we start from an analogy with quantum phase transitions in fermionic atomic gases or superconductors and consider the hypothesis [5,6,7] that the fermions of the Standard Model have tiny Lorentz-violating effects due to Fermi-point splitting (abbreviated FPS and explained below).

If Fermi-point splitting would indeed occur for the quarks and leptons of the Standard Model, then neutrinos may provide a *unique* window to the underlying theory [8,9,10,11]. Specifically, there could be new effects in neutrino oscillations, possibly showing significant **T** and **CP** violation (and perhaps even **CPT** violation). The aim of this talk is to sketch some of the potential FPS effects but we refer, in particular, to the contribution of M.C. González-García in these Proceedings for a more general discussion of nonstandard neutrino oscillations.

The outline of this write-up is as follows. In Sec. 2, some background on condensed matter physics is given and, in Sec. 3, a possible application to elementary particle physics is discussed. In Sec. 4, a simple but explicit neutrino model with both Fermi-point splittings and mass differences is introduced. In Sec. 5, some interesting results on neutrino oscillations from this model are reviewed. In Sec. 6, concluding remarks are presented.

2. Fermi-point splitting in atomic and condensed-matter systems

Ultracold quantum gases of fermionic atoms (e.g., ⁶Li at nano-Kelvin temperatures) are extremely interesting systems, especially as they can have *tunable* interactions by way of magnetic-field Feshbach resonances. In the so-called BEC–BCS

crossover region of these systems, a BCS-type condensate has recently been observed for s -wave pairing [12]. As usual, BEC stands for Bose–Einstein condensate and BCS for the superconductivity triumvirate Bardeen, Cooper, and Schrieffer.

For the BEC–BCS crossover region in systems with p -wave pairing, there is the prediction [5,6] that a *quantum phase transition* between a vacuum state with fully-gapped fermionic spectrum and a vacuum state with topologically protected Fermi points (gap nodes) occurs. Here, we only give a simple illustration of this new type of quantum phase transition and refer the reader to Ref. [13] for an extensive review.

The Bogoliubov–Nambu Hamiltonian for fermionic quasiparticles in the axial state of p -wave pairing is given by

$$H_{\text{BN}} = \begin{pmatrix} |\mathbf{p}|^2/(2m) - q & c_{\perp} \mathbf{p} \cdot (\hat{\mathbf{e}}_1 + i\hat{\mathbf{e}}_2) \\ c_{\perp} \mathbf{p} \cdot (\hat{\mathbf{e}}_1 - i\hat{\mathbf{e}}_2) & -|\mathbf{p}|^2/(2m) + q \end{pmatrix}, \quad (1)$$

with m the mass of the fermionic atom (considered is the direction of atomic spin, which experiences the Feshbach resonance), $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{l}})$ an orthonormal triad, $\hat{\mathbf{l}}$ the direction of the orbital momentum of the pair, c_{\perp} the maximum transverse speed, and q a parameter controlled by the magnetic field near the Feshbach resonance.

The energy spectrum of this Hamiltonian is readily calculated:

$$E_{\text{BN}}^2(\mathbf{p}) = (|\mathbf{p}|^2/(2m) - q)^2 + c_{\perp}^2 |\mathbf{p} \times \hat{\mathbf{l}}|^2. \quad (2)$$

Clearly, there are two regimes. For parameter $q < 0$, on the one hand, there is a BEC regime with mass gap, $E \neq 0$. For parameter $q > 0$, on the other hand, there is a BCS regime with two Fermi points in momentum space,

$$\mathbf{b}_1 = +p_F \hat{\mathbf{l}}, \quad \mathbf{b}_2 = -p_F \hat{\mathbf{l}}, \quad p_F \equiv \sqrt{2mq}, \quad (3)$$

at which the energy function vanishes, $E(\mathbf{p}) = 0$ for $\mathbf{p} = \mathbf{b}_a$ with $a = 1, 2$.

There is then a quantum phase transition at $q = 0$, with a mass gap for $q < 0$ and a spacelike splitting of Fermi points ($\Delta \mathbf{b} \equiv \mathbf{b}_1 - \mathbf{b}_2 \neq 0$) for $q > 0$; see Fig. 1.

This example also clarifies the concept of emergent relativity mentioned in the Introduction. Consider momenta close to one of the two Fermi points, for example, $\mathbf{p} = \mathbf{b}_1 + \mathbf{k}$ with $|\mathbf{k}| \ll p_F$. Then, the energy (2) becomes

$$E_{\text{BN}}^2 \sim (p_F/m)^2 k_{\parallel}^2 + c_{\perp}^2 k_{\perp}^2 \sim \tilde{c}^2 (\tilde{k}_{\parallel}^2 + \tilde{k}_{\perp}^2), \quad (4)$$

after the following rescalings:

$$k_{\parallel} \equiv \mathbf{k} \cdot \hat{\mathbf{l}} \equiv (\tilde{c}m/p_F) \tilde{k}_{\parallel}, \quad k_{\perp} \equiv |\mathbf{k} \times \hat{\mathbf{l}}| \equiv (\tilde{c}/c_{\perp}) \tilde{k}_{\perp}, \quad (5)$$

which would be appropriate for a local observer made of the *same* quasi-particles [4,13]. In terms of the rescaled momentum $\tilde{\mathbf{k}}$, relation (4) corresponds precisely to the mass-shell condition of a massless relativistic particle.

3. FPS hypothesis for elementary particle physics

Based on the analogy with certain condensed-matter systems discussed in Sec. 2, the following hypothesis has been put forward [5,6]: perhaps the chiral fermions of the Standard Model also have Fermi-point splitting (FPS). Specializing to time-like splittings ($\Delta b_0 \neq 0$) and vanishing Yukawa coupling constants (i.e., vanishing fermion masses), their dispersion relations would be given by:

$$(E_{a,f}(\mathbf{p}))^2 = \left(c |\mathbf{p}| + b_{0a}^{(f)} \right)^2, \quad (6)$$

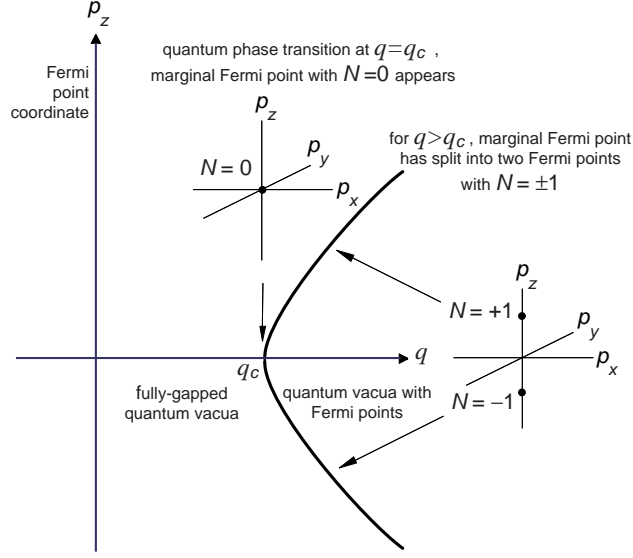


Fig. 1. Quantum phase transition at $q = q_c$ between a quantum vacuum with mass gap and one with topologically-protected Fermi points (gap nodes). At $q = q_c$, there appears a marginal Fermi point with topological charge $N = 0$ (inset at the top). For $q > q_c$, the marginal Fermi point has split into two Fermi points characterized by nonzero topological invariants $N = \pm 1$ (inset on the right). A system described by Hamiltonian (1), for $\hat{\mathbf{l}} = (0, 0, 1)$, has critical parameter $q_c = 0$.

where a labels the 16 types of massless left-handed Weyl fermions (including a left-handed antineutrino) and f the N_{fam} fermion families (henceforth, we take $N_{\text{fam}} = 3$). The maximum velocity of the fermions is assumed to be universal and equal to the velocity of light *in vacuo*, c . Note that we still speak about Fermi-*point* splitting even though the energy (6) for $b_{0a}^{(f)} < 0$ gives rise to a Fermi *surface*.

One possible FPS pattern is given by the following *factorized* Ansatz [6]:

$$b_{0a}^{(f)} = Y_a \tilde{b}_0^{(f)}, \quad (7)$$

where Y_a are the known hypercharge values of the fermions and $\tilde{b}_0^{(f)}$ three unknown energy scales. Independent of the particular FPS pattern, the dispersion relations of massless left-handed neutrinos and right-handed antineutrino would be

$$(E_{\nu_{L,f}}(\mathbf{p}))^2 = (c|\mathbf{p}| + b_0^{(f)})^2, \quad (E_{\bar{\nu}_{R,f}}(\mathbf{p}))^2 = (c|\mathbf{p}| + s b_0^{(f)})^2, \quad (8)$$

where a value $s = 1$ respects CPT and $s = -1$ violates it.

More generally, one may consider for large momentum $|\mathbf{p}|$:

$$E(\mathbf{p}) \sim c|\mathbf{p}| \pm b_0 + m^2 c^4 / (2c|\mathbf{p}|) + \mathcal{O}(1/|\mathbf{p}|^2). \quad (9)$$

The conclusion is then that the search for possible FPS effects prefers neutrinos with the highest possible momentum.

At this point, two questions on energy scales arise. First, what is known experimentally? The answer is: not very much, apart from the following upper bounds:

$$|b_0^{(e)}| \lesssim 1 \text{ keV}, \quad \sum_{f=1}^3 m_f \lesssim 100 \text{ eV}, \quad (10)$$

from low-energy neutrino physics [14] and cosmology, respectively.

Second, what can be said theoretically about the expected energy scale of FPS? The answer is: little to be honest, but perhaps the following speculation may be of some value. For definiteness, start from a particular emergent-physics scenario with two energy scales [7]:

- E_{LV} of the fundamental Lorentz-violating fermionic theory;
- E_{comp} as the compositeness scale of the Standard Model gauge bosons.

Taking the LEP values of the gauge coupling constants, the renormalization-group equations for $N_{\text{fam}} = 3$ give these numerical values:

$$E_{\text{comp}} \sim 10^{13} \text{ GeV}, \quad E_{LV} \sim 10^{42} \text{ GeV}. \quad (11)$$

The speculation, now, is that perhaps ultrahigh-energy Lorentz violation *re-enters* at an ultralow energy scale:

$$|b_0| \stackrel{?}{\sim} E_{\text{comp}}^2 / E_{LV} \sim 10^{-7} \text{ eV}. \quad (12)$$

If correct, this motivates the search for FPS effects at the sub-eV level.

4. Simple FPS neutrino model

A general neutrino model with *both* Fermi-point splittings (FPS) and mass differences (MD) has many mixing angles and complex Dirac phases to consider (not to mention possible Majorana phases). In order to get an idea of potentially new effects, consider a relatively simple FPS–MD neutrino model [10,11] having

- a standard neutrino mass sector with “optimistic” values for θ_{13} and δ ;
- a FPS sector with large mixing angles, energy splittings, and Dirac phase ω .

Specifically, the mass sector has the following mass-square-difference ratio, mixing angles, and Dirac phase:

$$R_m \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \equiv \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} = \frac{1}{30}, \quad \theta_{21} = \theta_{32} = \frac{\pi}{4}, \quad \sin^2 2\theta_{13} = \frac{1}{20}, \quad \delta = \frac{\pi}{2}, \quad (13a)$$

and the FPS sector has energy-difference ratio, mixing angles, and Dirac phase:

$$R \equiv \frac{\Delta b_0^{(21)}}{\Delta b_0^{(32)}} \equiv \frac{b_0^{(2)} - b_0^{(1)}}{b_0^{(3)} - b_0^{(2)}} = 1, \quad \chi_{21} = \chi_{32} = \chi_{13} = \omega = \frac{\pi}{4}. \quad (13b)$$

For later use, we also define two additional models. The first additional model is a *pure* FPS model [9] with trimaximal couplings ($\chi_{21} = \chi_{32} = \pi/4$ and $\chi_{13} = \arctan \sqrt{1/2}$), complex phase ω , and FPS ratio R . At sufficiently high energies, the model for $R = 1$ and $\omega = \pi/4$ is close to the FPS–MD model mentioned above.

The second additional model is a *pure* MD model with a mass-square-difference ratio $R_m = 1/30$ and the following more or less realistic values for the mixing angles and Dirac phase: $\sin^2 2\theta_{23} = 1$, $\sin^2 2\theta_{12} = 0.8$, $\sin^2 2\theta_{13} = 0.2$, and $\delta = 0$.

In the rest of this contribution, these three models will be referred to as the FPS–MD model, the FPS model, and the MD model, respectively.

5. FPS effects in neutrino oscillations

Consider now a high-energy ($E_\nu \sim c|\mathbf{p}|$) neutrino beam traveling over a distance L . Neutrino oscillations from the FPS–MD model of Sec. 4 are then determined by two dimensionless parameters:

$$\rho \equiv \frac{2 E_\nu \hbar c}{L |\Delta m_{31}^2| c^4} \approx 0.98 \left(\frac{E_\nu}{20 \text{ GeV}} \right) \left(\frac{3000 \text{ km}}{L} \right) \left(\frac{2.7 \times 10^{-3} \text{ eV}^2 / c^4}{|\Delta m_{31}^2|} \right), \quad (14a)$$

$$\{R_m, \sin^2(2\theta_{13}), \delta, \text{sgn } \Delta m^2; R, \sin^2(2\chi_{13}), \omega, \text{sgn } \Delta b_0\} = \{1/30, 1/20, \pi/2, +; 1, 1, \pi/4, +\}$$

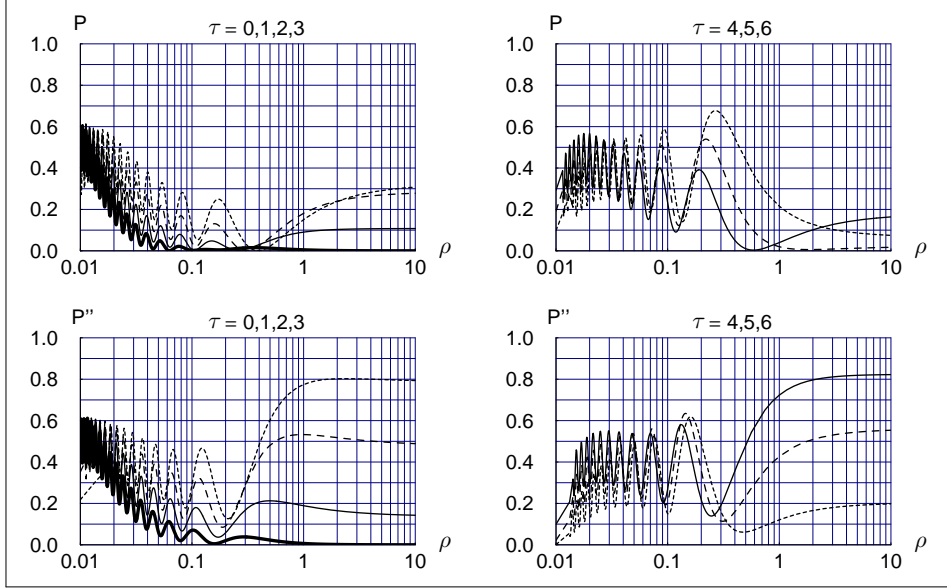


Fig. 2. Vacuum probabilities from the FPS–MD model (13ab) as a function of the dimensionless parameters ρ and τ , defined by Eqs. (14ab). Top panels: $P \equiv P(\nu_\mu \rightarrow \nu_e)$. Bottom panels: $P'' \equiv P(\nu_e \rightarrow \nu_\mu)$. If CPT invariance holds, also $P = P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ and $P'' = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$. Shown are constant- τ slices, where the heavy-solid curves in the two left panels correspond to $\tau = 0$ (pure mass-difference model) and the other thin-solid, long-dashed, and short-dashed curves for positive τ correspond to $\tau = 1, 2, 0 \pmod{3}$, respectively.

$$\tau \equiv L |\Delta b_0^{(31)}| / (\hbar c) \approx 3.0 \left(\frac{L}{3000 \text{ km}} \right) \left(\frac{|\Delta b_0^{(31)}|}{2.0 \times 10^{-13} \text{ eV}} \right). \quad (14b)$$

for numerical values of L and E_ν appropriate to a neutrino factory [15]. Possible new effects in neutrino oscillations from FPS may occur as

- energy dependence of the vacuum mixing angle Θ_{13} [10];
- novel source of T, CP, and perhaps CPT violation [11];
- modified flavor ratios for high-energy cosmic neutrinos [8,9].

In this contribution, we discuss only the last two effects.

Figure 2 shows that, provided the FPS parameter $\Delta b_0^{(31)}$ is large enough for given baseline L , the probabilities of time-reversed processes can be different by several tens of percents: $P(\nu_\mu \rightarrow \nu_e) \approx 20\%$ versus $P(\nu_e \rightarrow \nu_\mu) \approx 80\%$ at $\rho \sim 1$ and $\tau \sim 3$, for example. For the record, standard mass-difference neutrino oscillations ($\tau = 0$) give more or less equal probabilities at $\rho \sim 1$: $P(\nu_\mu \rightarrow \nu_e) \approx P(\nu_e \rightarrow \nu_\mu) \approx 0$. In short, there could be strong T-violating (and CP-violating) effects at the high-energy end of the neutrino spectrum from FPS or other emergent-physics dynamics.

Next, turn to the pure FPS model and also, for comparison, to the pure MD model, both defined in Sec. 4. Pion and neutron sources then give the averaged event ratios shown in Table 1, with the clearest difference between the two models for the case of a neutron source. In principle, these results may be relevant to high-energy cosmic neutrinos but it remains to be seen whether or not present experiments (e.g., AMANDA and IceCube) can access this type of information.

Table 1

Averaged event ratios ($N_e : N_\mu : N_\tau$) from pion and neutron sources for pure Fermi-point-splitting (FPS) and mass-difference (MD) neutrino models as defined in Sec. 4. The MD event ratios are taken from Ref. [16].

	π : initial ratios = (1 : 2 : 0)	n : initial ratios = (1 : 0 : 0)
FPS (ω)	(6 : 7 + $\cos 2\omega$: 5 - $\cos 2\omega$)	(1 : 1 : 1)
FPS ($\pi/4$)	(0.33 : 0.39 : 0.28)	(0.33 : 0.33 : 0.33)
MD	(0.36 : 0.33 : 0.31)	(0.56 : 0.26 : 0.18)

6. Outlook

From a phenomenological perspective, the Fermi-point-splitting (FPS) hypothesis suggests the following three research directions:

- the possible energy dependence of the vacuum mixing angle Θ_{13} from FPS, which can be tested by neutrino experiments at a superbeam or neutrino factory;
- the possibility of a new source of leptonic CP violation, which impacts on the physics of the early universe (e.g., the creation of baryon and lepton number);
- the possible modification of the propagation of high-energy cosmic neutrinos by FPS effects, which may be of relevance to present and future neutrino telescopes.

From a more theoretical perspective, the outstanding issues are:

- the precise nature of the conjectured re-entrance mechanism of Lorentz violation at ultralow energy from Lorentz violation at ultrahigh energy (condensed-matter physics can perhaps provide some guidance);
- the explanation of the large hierarchies of basic scales (e.g., for mass or FPS).

But apart from these theoretical ideas, experiment may, of course, suggest entirely different directions . . .

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